Turn in the following problems:

1. A 2018 Dodge Challenger SRT Demon accelerates from 0 to 88 ft/sec (60 mph) in 2.3 seconds. This qualified it as the fastest manufacturer-timed acceleration in 2018.
   (a) Assuming that acceleration is constant, graph the velocity from $t = 0$ to $t = 2.3$
   (b) How far does the car travel during this time?

2. Speedometer readings for a motorcycle at 12-second intervals are given in the table below.

t (s)	0	12	24	36	48	60
v (ft/s)	30	28	25	22	24	27

   (a) Estimate the distance traveled by the motorcycle during this time period using the velocities at the beginning of the time intervals.
   (b) Give another estimate using the velocities at the end of the time periods.
   (c) Are your estimates in parts (a) and (b) upper estimates? Are they lower estimates? Explain.

3. Fill in the blank with "all", "no", or "some" to make the following statements true. Note that "some" means one or more instances, but not all.
   - If your answer is "all", then give a brief explanation as to why.
   - If your answer is "no", then give a brief explanation as to why.
   - If your answer is "some", then give two specific examples that illustrate why your answer it not "all" or "no".

   An example must include either a graph or a specific function.

   (a) For _______ functions f, if f has an antiderivative, then this antiderivative is unique.
   (b) For _______ functions f, if $f(x)$ is a linear function with positive slope, then any antiderivative of f is also a linear function.
   (c) For _______ antiderivatives F of a given function f, if $f(x)$ is positive over an interval, then $F(x)$ is also positive over that interval.
   (d) For _______ functions f, if $f(x)$ is increasing over an interval, then every antiderivative $F(x)$ is concave up over that interval.

   In mathematics, we consider a statement to be false if we can find any examples where the statement is not true. We refer to these examples as counterexamples. Note that a counterexample is an example for which the "if" part of the statement is true, but the "then" part of the statement is false.

4. Find the most general antiderivative of the function. (Check your answer by differentiation.)

$$f(x) = \frac{2 + x^2}{1 + x^2}$$

$\boxed{\text{Let } g(x) = 1 - x \text{ and } f(x) = x^2 - 2x + 1 \text{ for Problems 5 and 6 below.}}$

5. (a) Draw a graph of $f(x)$ and $g(x)$ on the same axes, and label their points of intersection. Calculate the area below $g(x)$ and above the x-axis between $x = 0$ and $x = 1$ using geometric methods. How does the area under f between $x = 0$ and $x = 1$ compare to the area under g?

   (b) Use 4 rectangles to calculate L_4, the area estimate under $f(x)$ between $x = 0$ and $x = 1$ using left endpoints. Then calculate R_4, the area estimate using right endpoints.

   (c) If A is the true area under $f(x)$ between $x = 0$ and $x = 1$, compare L_4, R_4, and A.

6. We will use the following definition to calculate the exact area under $f(x)$ from $x = 0$ to $x = 1$.

   Definition: *The* **area** *A of the region S that lies under the graph of the continuous function f is the limit of the sum of the areas of approximating rectangles:*

$$\text{Area } = A = \lim_{n\to\infty} R_n = \lim_{n\to\infty} [f(x_1)\Delta x + f(x_2)\Delta x + \cdots + f(x_n)\Delta x]$$

   (a) Use the definition of area given above to write a limit that is equal to the area under the graph of f from $x = 0$ to $x = 1$. Do not evaluate the limit yet.

   (b) Evaluate the limit in part (a) using these two formulas, both found in Theorem 3 of Appendix F of your book.

$$1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6} \qquad 1 + 2 + 3 + \cdots + n = \frac{n(n+1)}{2}$$

   (c) The answer for part (b) should represent the area under $f(x)$ between $x = 0$ and $x = 1$. Explain briefly why this limit represents the area under f.

These problems will not be collected, but you might need the solutions during the semester:

7. Two balls are thrown upward from the edge of the cliff 432 ft above the ground. The first is thrown at a speed of 48 ft/s and the other is thrown a second later with a speed of 24 ft/s. Do the balls ever pass each other?

8. As the flu spreads through a population, the number of infected people, I, is expressed by a function of the number of susceptible people S, by

$$I = k\ln\left(\frac{S}{S_0}\right) - S + S_0 + I_0, \text{ for } k, S_0, I_0 > 0$$

   (a) Find the maximum number of infected people.

   (b) The constant k is a characteristic of the particular strain of the flu; the constants S_0 and I_0 are the values of S and I when the flu starts. Which of the following affects the maximum possible value of I? Explain

   - The particular strain of flu, but now how it starts.
   - How the disease starts, but not the particular strain of flu.
   - Both the particular strain of the flu and how it starts.

9. (a) Batman was driving the Batmobile at 90 mph (=132 ft/sec), when he sees a brick wall directly ahead. When the Batmobile is 400 ft from the wall, he slams on the brakes, decelerating at a constant rate of 22 ft/s^2. Does he stop before he hits the brick wall? If so, how many feet to spare? If not, what is his impact speed?

   (b) Now the Joker had been driving next to Batman, also at 90 mph. But the Joker did not hit his brakes as soon as Batman, continuing for 1 second longer than Batman before hitting his brakes, decelerating at a constant rate of 22 ft/s^2. How fast is he going when he hits the wall? (Don't worry about Joker - he jettisoned at the last instant, to fight for another day!)